MATH 1450 EXAM 4

NAME Key — GRADE ______ OUT OF **15 PTS**

Answer each of the next questions separately and **show** your correct work for a full credit.

1. **(3pts) Antiderivatives:** u**-substitution (3pts)** -SHOW ALL STEPS for a all full credit

   (a) $\displaystyle\int \frac{1}{x\ln x}\,dx$

   (b) $\displaystyle\int \frac{x}{(8-2x^2)^3}\,dx$

   (c) $\displaystyle\int (4+3x)^4(3)\,dx$

(a). Let $u = \ln x$ $du = \frac{1}{x}dx$ so we have $\int \frac{1}{u} \cdot du$

$= \ln|u| = \ln[|\ln(x)|] + c$

(b) $u = 8-2x^2 \longrightarrow du = -4x\,dx \longrightarrow -\frac{1}{4}du = x\,dx$

So $\int \frac{1}{u^3}\left(-\frac{1}{4}du\right) = -\frac{1}{4}\int u^{-3}du = -\frac{1}{4}\left(\frac{-1}{2}u^{-2}\right] + c$

$= \frac{1}{8u^2} + c = \frac{1}{8(8-2x^2)^2} + C$

(c) $u = 4+3x \longrightarrow du = 3x\,dx \longrightarrow$

$\int u^4 du \longrightarrow \frac{1}{5}u^5 + c \longrightarrow \frac{1}{5}(4+3x)^5 + c$

2. **Definite integrals and FTC (3pts)** (*No decimal* answer!)

(a) if $y = \int_0^{\sqrt{x}} t^3\sqrt{1+t^2}\,dt$, find y'

(b) Evaluate $\int_0^{\ln(3)} 2e^x\,dx$.

(c) Evaluate $\int_0^1 \frac{x-\sqrt{x}}{6}\,dx$

(a) $y' = \frac{d}{dx}\left[\int_0^{\sqrt{x}} t^3\left(\sqrt{1+t^2}\right)dt\right]$

$= (\sqrt{x})^3\sqrt{1+[\sqrt{x}]^2}\cdot\frac{d}{dx}[\sqrt{x}]$

$= x^{3/2}\left(\sqrt{1+x}\right)\left(\frac{1}{2\sqrt{x}}\right)$ or $\frac{x\sqrt{x}}{2\sqrt{x}}\left(\sqrt{1+x}\right)$

or $\boxed{\frac{x}{2}\sqrt{1+x}}$

(b) $2\int_0^{\ln 3} e^x dx = 2\left[e^x\right]_0^{\ln 3} = 2e^{\ln 3} - 2e^0$

$= 2(3) - 2(1) = \boxed{4}$

(c) $\frac{1}{6}\int_0^1 (x-\sqrt{x})\,dx = \frac{1}{6}\left[\frac{1}{2}x^2 - \frac{2}{3}x^{3/2}\right]_0^1$

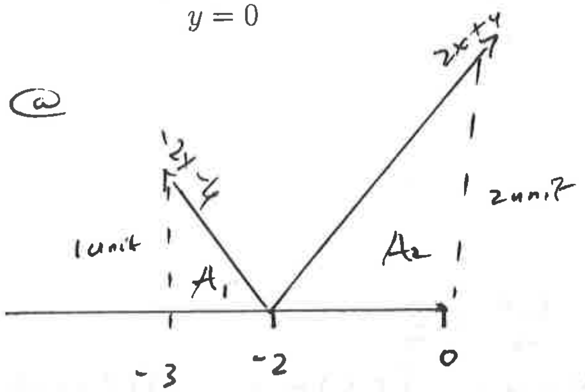
$= \frac{1}{6}\left[+\frac{1}{2} - \frac{2}{3} - 0\right] = \frac{1}{6}\left(\frac{-1}{6}\right) = \boxed{\frac{-1}{36}}$

3. **Area under the curve and FTC (3pts)** (*No decimal* answer!)

(a) Calculate the area above the x-axis, and below the graph of $g(x) = |2x+4|$ which is bounded by $x = -3$ and $x = 0$ using:
(i) geometry (ii) integration (**show** your work!).

(b) Find the area of the region bounded by the graphs of the equations. $y = \frac{1}{x}$, $x = 1$, $x = e$, $y = 0$

(a)



(i) $\text{Area} = A_1 + A_2$

$= \frac{1}{2}(1)(2) + \frac{1}{2}(2)(4)$

$= 1 + 4 = \boxed{5 \text{ unit}^2}$

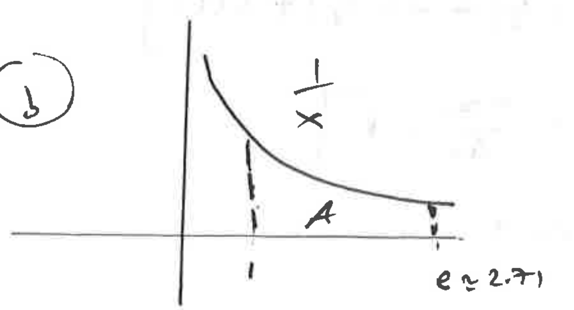
(ii) $\int_{-3}^{-2} -(2x+4)\,dx + \int_{-2}^{0} (2x+4)\,dx$

$= -2\left[+\frac{1}{2}x^2 + 4x\right]_{-3}^{-2} + 2\left[\frac{1}{2}x^2 + 2x\right]_{-2}^{0}$

$= 1 + 4$

$\boxed{= 5 \text{ unit}^2}$

(b)



$A = \int_1^e \frac{1}{x}\,dx = \left[\ln|x|\right]_1^e$

$= \ln e - \ln 1$

$= 1 - 0$

$\boxed{A = 1 \text{ unit}^2}$

4. **Area approximation: Riemann Sums** (5pts) (up to *four* decimals answer!)

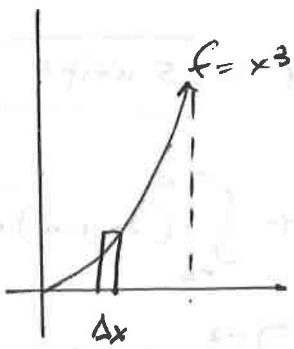
(a) Use a **Left** Sum with **four** rectangles of equal width to approximate the area under the graph of $f(x) = x^3$, between $x = 0$ and $x = 1$.

(b) Use a **Right** Sum with **four** rectangles of equal width to approximate the area under the graph of $f(x) = x^3$, between $x = 0$ and $x = 1$.

(c) From the previous answers in (a) and (b), what would be the area of the region using a Trapezoidal rule?

(d) What is the exact value of the region? (use FTC rule)

(e) What is the percentage of error when you compare the value found in (c) to the one found in (d)?



base ; $\Delta x = \frac{1-0}{4} = \frac{1}{4}$

heights: $f(0) = 0$; $f(\frac{1}{2}) = \frac{1}{8}$, $f(\frac{1}{4}) = \frac{1}{64}$

$f(\frac{3}{4}) = \frac{27}{64}$; $f(1) = 1$

(a) L_4 : $\frac{1}{4}\left[f(0) + f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4})\right]$

$= \frac{1}{4}\left[0, + \frac{1}{64} + \frac{1}{8} + \frac{27}{64}\right]$

$L_4 \simeq .1406$ unit²

(b) $R_4 = \frac{1}{4}\left[f(\frac{1}{4}) + f(\frac{1}{2}) + f(\frac{3}{4}) + f(1)\right]$

$= \frac{1}{4}\left[\frac{1}{64} + \frac{1}{8} + \frac{27}{64} + 1\right]$

$\simeq .3906$ unit²

(c) $T \doteq \frac{L_4 + R_4}{2} \simeq \frac{(.1402 + .3906)}{2} \simeq .2656$

(d) $\int_0^1 x^3 dx = \frac{1}{4}\left[x^4\right]_0^1 = \frac{1}{4}[1-0] = \frac{1}{4} \doteq .25$

(e) Error : $\left| .25 - .2656 \right| \simeq .0156$

$\frac{\text{Error}}{.25} \times 100 \longrightarrow$ percentage: 6.256 % of error